

# Design optimal Fractional PID Controller for DC Motor with Genetic Algorithm

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**Abstract**— An intelligent optimization method for designing Fractional Order PID (FOPID) controller's base Genetic Algorithm (GA) ear presented in this paper. Fractional calculus can provide novel and higher performance extension for FOPID controllers. However, the difficulties of designing FOPID controllers increase, because FOPID controllers append derivative order and integral order in comparison with traditional PID controllers. To design the parameters of FOPID controllers. Experimental results show the proposed design method can design effectively the parameters of FOPID controllers.

**Index Terms**— Genetic Algorithm; Fractional order PID; fractional calculus; PID control; DC Motor.

## 1 INTRODUCTION

Fractional order control systems are described by fractional order differential equations. Fractional calculus allows the derivatives and integrals to be any real number. The FOPID controller is the expansion of the conventional PID controller based on fractional calculus. FOPID controllers' parameters designed have five, and the derivative and integral orders improve the design flexibility.

### 1.1 FRACTIONAL CALCULUS

There are several definitions of fractional derivative [1]. Grunwald-Letnikov definition is perhaps the best known one due to its most suitable for the realization of discrete control algorithms. The m order fractional derivative of continuous function f(t) is given by:

$$D^m f(t) = \lim_{h \rightarrow 0} h^{-m} \sum_{j=0}^{[x]} (-1)^j \binom{m}{j} f(t - jh) = \frac{d^m f(t)}{dt^m} \quad (1)$$

Where [x] is a truncation and  $x = \frac{t-m}{h}$ ;  $\binom{m}{j}$  is binomial coefficients,

$$\binom{m}{j} = \frac{m(m-1) \dots (m-j+1)}{j!} \quad (2)$$

$\binom{m}{j} = 1, (j = 0)$ , it can be replaced by Gamma function,  $\binom{m}{j} = \frac{\Gamma(m+1)}{j! \Gamma(m-j+1)}$

The general calculus operator, including fractional order and integer, is defined as:

$$D_t^\alpha = \begin{cases} \frac{d^\alpha}{dt} & R(\alpha) > 0 \\ 1 & R(\alpha) = 0 \\ \int_0^t (d\tau)^{-\alpha} & R(\alpha) < 0 \end{cases} \quad (3)$$

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$$\mathcal{L}\{D^\alpha f(t)\} = s^\alpha F(s) - [D^{\alpha-1} f(t=0)] \quad (4)$$

Where f(s) is the Laplace transform of f(t) The Laplace transform of the fractional integral of f(t) is given as follows:

$$\mathcal{L}\{D^{-\alpha} f(t)\} = s^{-\alpha} F(s) \quad (5)$$

### 1.2 FRACTIONAL ORDER CONTROLLERS

The differential equation of fractional order controller  $PI^\alpha D^\beta$  is described by [2]:

$$u(t) = K_p e(t) + K_i D_t^{-\lambda} e(t) + K_d D_t^\delta e(t) \quad (6)$$

The continuous transfer function of FOPID is obtained through Laplace transform, which is given by:

$$u(t) = K_p e(t) + K_i D_t^{-\lambda} e(t) + K_d D_t^\delta e(t) \quad (7)$$

$$G_c(s) = K_p + K_i s^{-\lambda} + K_d s^\delta \quad (8)$$

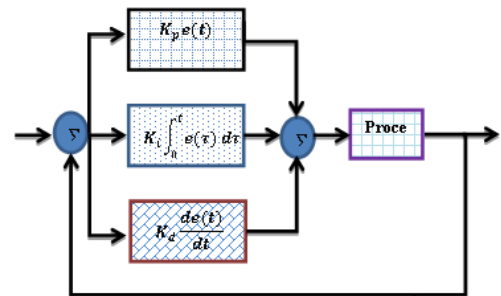


Fig. 1. Generic closed loop control system With a PID controller

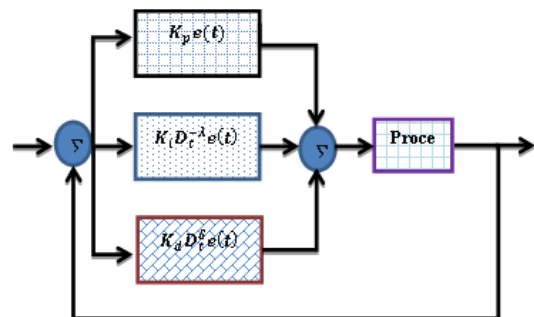


Fig. 2. Generic closed loop control system With a FOPID controller

It is obvious that the FOPID controller not only need design three parameters, Kp, Ki and, Kd but also design two orders, λ, δ of integral and derivative controllers. The orders, λ, δ are not necessarily integer, but any real numbers. As shown in Fig.3 the FOPID controller generalizes the conventional integer order PID controller and expands it from point to plane. This expansion could provide much more flexibility in PID control design. [3]

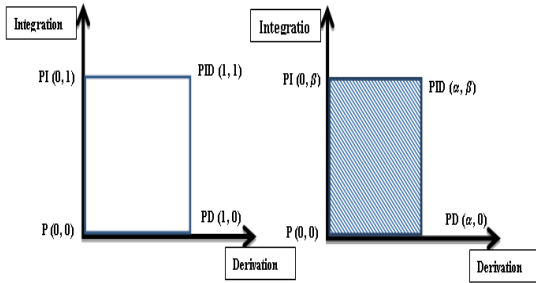


Fig. 3. PID controllers with fractional orders

**2 MODEL OF DC MOTOR**

DC machines are characterized by their versatility. By means of various combinations of shunt-, series-, and separately-excited field windings they can be designed to display a wide variety of volt-ampere or speed-torque characteristics for both dynamic and steady-state operation. Because of the ease with which they can be controlled systems of DC machines have been frequently used in many applications requiring a wide range of motor speeds and a precise output motor control [3,4].

In this paper, the separated excitation DC motor model is chosen according to his good electrical and mechanical performances more than other DC motor models. The DC motor is driven by applied voltage. Fig.4 show the equivalent circuit of DC motor with separate excitation. The Symbols, Designations and Units are publicized in Table 1.

The characteristic equations of the DC motor are represented as:

$$\frac{d}{dt} i_{ex} = \left(-\frac{R_{ex}}{L_{ex}}\right) \cdot i_{ex} + \left(\frac{1}{L_{ex}}\right) \cdot V_{ex} \quad (1)$$

$$\frac{d}{dt} i_{ind} = \left(-\frac{R_{ind}}{L_{ind}}\right) \cdot i_{ind} + \left(-\frac{L_{index}}{L_{ind}}\right) \cdot W_r \cdot I_{ex} + \left(\frac{1}{L_{ind}}\right) \cdot V_{ind} \quad (2)$$

$$\frac{d}{dt} W_r = \left(-\frac{L_{index}}{J}\right) \cdot i_{ex} \cdot i_{ind} + \left(-\frac{Cr}{J}\right) \cdot W_r \cdot I_{ex} + \left(\frac{-fc}{J}\right) \cdot W_r \quad (3)$$

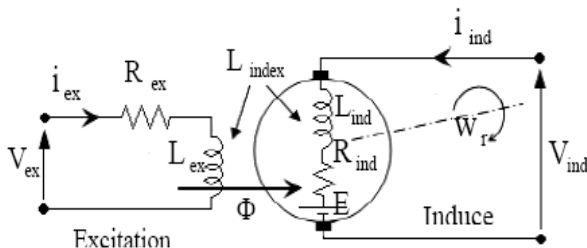


Fig. 4: PID controllers with fractional orders

TABLE 1  
SYMBOLS, DESIGNATIONS AND UNITS

Symbols	Designations	Units
i <sub>ex</sub> and i <sub>ind</sub>	Excitation current and Induced current.	[A]
w <sub>r</sub>	Rotational speed of the DC Motor.	[Rad/Sec]
V <sub>ex</sub> and V <sub>ind</sub>	Excitation voltage and Induced voltage	[Volt]
R <sub>ex</sub> and R <sub>ind</sub>	Excitation Resistance and Induced Resistance.	[Ω]
L <sub>ex</sub> , L <sub>ind</sub> and L <sub>index</sub>	Excitation Inductance Induced Inductance and Mutual Inductance.	[mH]
J	Moment of Inertia.	[Kg.m <sup>2</sup> ]
Cr	Couple resisting.	[N.m]
fc	Coefficient of Friction.	[N.m.Sec/Rad ]

From the state equations (1), (2), (3) previous, can construct the model with the environment MATLAB. The model of the DC motor in Simulink is shown in Fig. 5. The various parameters of the DC motor are shown in Table 2.

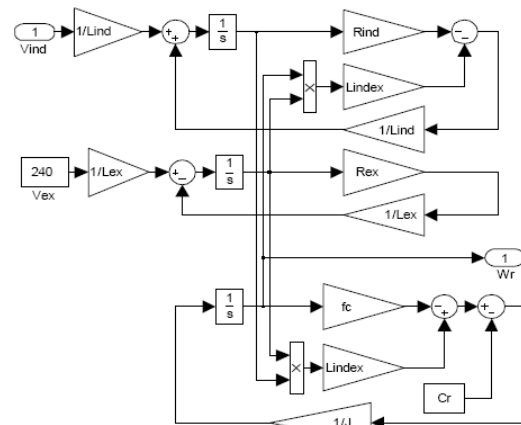


Fig.5: Model of the DC Motor in Simulink

TABLE 2 .PARAMETERS OF THE DC MOTOR

V <sub>ex</sub> =240[V]	L <sub>ind</sub> =0.012[mH]
V <sub>ind</sub> =240[V]	L <sub>index</sub> =1.8[mH]
R <sub>ex</sub> =240[Ω]	J=1[Kg.m <sup>2</sup> ]
R <sub>ind</sub> =0.6[Ω]	Cr=29.2[N.m]
L <sub>ex</sub> =120[mH]	fc=0.0005[N.m.Sec/Rad]

Into the expressions for N and P. Rather than continuing with algebra here, we will simply represent these equations in Simulink.

Simulink can work directly with nonlinear equations

### 3 GA BASED TUNING OF THE $PI^{-\lambda} D^{\delta}$ CONTROLLER GAINS

#### Introduction to Genetic Algorithm $PI^{-\lambda} D^{\delta}$

In 1975, GA was proposed firstly by Holland [5]. It is an optimization algorithm and applied to various fields, including business, science, and engineering. Based on the survival-of the-fittest strategy proposed by Darwin, this algorithm will eliminate unfit components to select the fittest component by Man-made fitness functions generation by generation.

#### A. Initialization

In the initialization, the first thing to do is to decide the coding structure. Coding for a solution, termed a chromosome in GA literature, is usually described as string of symbols from  $\{0, 1\}$ . These components of the chromosome are then labeled as genes. The number of bits that must be used to describe the parameters is problem dependent.

#### B. Selection

GA uses proportional selection; the population of next generation is determined by n independent random experiments.

#### C. Crossover

Cross over is an important random operator in GA and the function of this operator is to generate a new 'child' chromosome from two 'parents' chromosomes by combining the information extracted from the parents.

#### D. Mutation

Mutation is another important component in GA, though it is usually conceived as a background operator. It operates independently on each individual by probabilistically perturbing each bit string. A usual way of mutation used in GA is to generate a random number between zero and one and then make a random change in the v-th element of the string with probability  $p_m$  belonging to  $(0, 1)$ .

#### E. Encoding & Decoding

The design variables are mapped onto a fixed-length binary Digit string, which are constructed over the binary alphabet  $\{0,1\}$ , and is concatenated head-to-tail to form one long string referred as a chromosome. That is, every string contains all design variables. The physical values of the design variables are obtained by decoding the string.

#### F. Fitness Function

In GA, the value of fitness represents the performance, which is used to rank the string, and the ranking is used to determine how to allocate reproductive opportunities. This means that individuals with higher fitness value will have higher probability of selection as a parent. Fitness thus is some measure of goodness to be optimized. The fitness function is essentially the objective function for the problem.

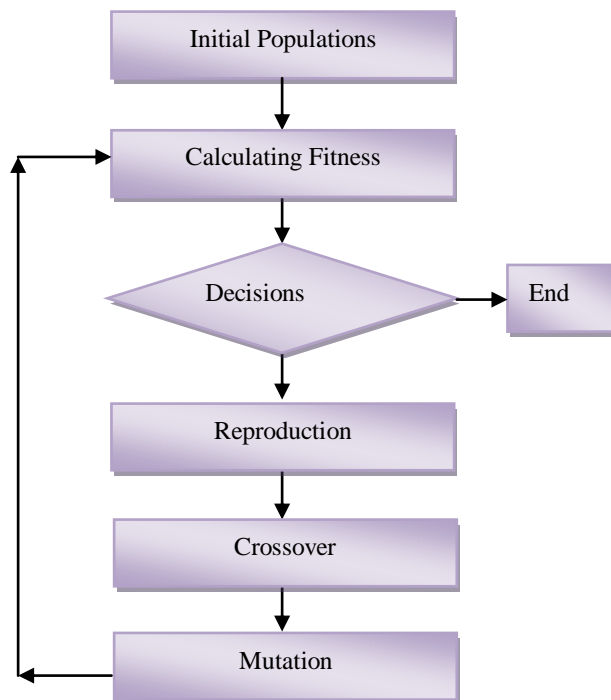


Fig.6: illustrates the block structure of the FOPID controller optimizing process with GA

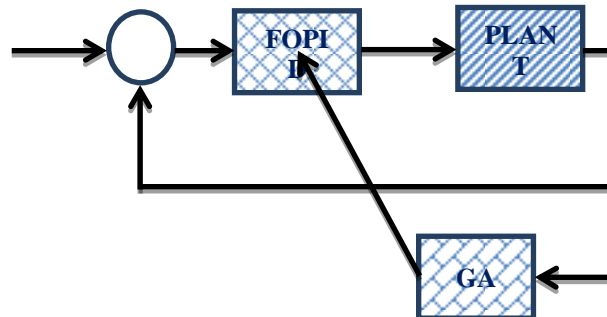


Fig.7: Tuning process of the FOPID controller parameters with GA

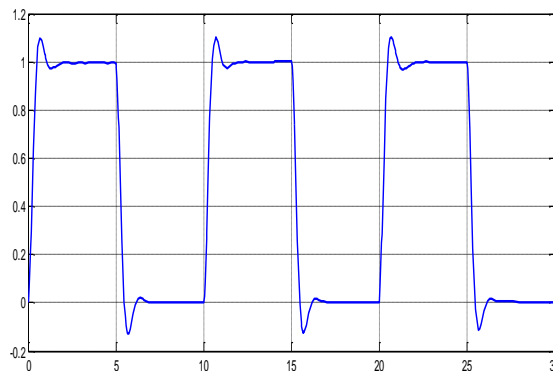


Fig.8: PID controller with GA

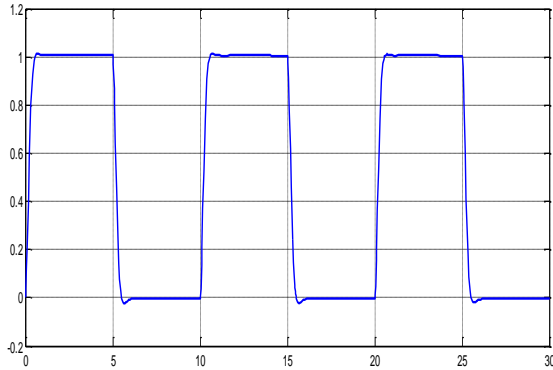


Fig.9: FOPID controller with GA

## 4 CONCLUSION

It has been demonstrated that the parameters optimization of fractional order controller based on modified GA is highly effective. According to optimization target, the proposed method can search the best global solution for FOPID controllers' parameters and guarantee the objective solution space in defined search space. Based on improved GA, the design and application of FOPID will be appeared in various fields.

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